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COMPUTATIONAL RESEARCH DIVISION

Generalized Utility Metrics for Supercomputers

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How to rank Supercomputer?

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- The TOP500 is often criticized for its simplicity.
- What influences the usefulness of our Supercomputers these days?
  - Performance (P)
  - Memory size (M)
  - Power Consumption (Power)
  - Floor Space (maybe later)
  - $: Best, but way too ‘touchy’!
- Nobody can select an appropriate performance measure for you!

Generalized Ranking

- **We want an “*extensive*” “Utility Metric” for Supercomputers to rank them.**
- **We should ask ourselves:** ***When is a Supercomputer x times as powerful as my current one?***
  - **Let’s focus on:**
    - **Performance (P)**
      - **(Computational, I/O, etc)**
    - **Memory size (M)**
      - **Disk size etc**
    - **Power Consumption (Power)**
    - **Floor Space (Space)**

Generalized Utility Metric

- Let's construct a Utility Metric (UM).
- The influence of these extra factors depend on your situation.
  - Include Weights for them.
  - The size of these weights will (to some extent) depend on you.
- Relative ranks have to independent of scales:

$$UM(a * x) = f(a) * UM(x)$$

- Simplest functional form is a weighted geometric mean:

$$UM(x_i) = \prod x_i^{\alpha_i}$$

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Generalized Utility Metric

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- **We want something which scales in first order like *performance* (extensive) with corrections from the other features (intensive):**

$$UM(SC) = P^{\alpha} * \left(\frac{P}{Peak}\right)^{\beta} * \left(\frac{M}{Peak}\right)^{\gamma} * \left(\frac{Peak}{Power}\right)^{\delta} * \left(\frac{Peak}{Space}\right)^{\epsilon}$$

$$\alpha, \beta, \gamma, \delta, \epsilon \geq 0$$

Generalized Utility Metric

- **Scale Peak is arbitrary and UM has to be independent from it:**

$$\beta + \gamma = \delta + \epsilon$$

$$UM(SC) = P^{\alpha} * \left(\frac{M}{P}\right)^{\gamma} * \left(\frac{P}{Power}\right)^{\delta+\epsilon} * \left(\frac{Power}{Space}\right)^{\epsilon}$$

Generalized Utility Metrics for Supercomputers

- **Scale Peak is arbitrary and UM has to be independent from it:** $\beta + \gamma = \delta + \epsilon$

$$UM(SC) = P^{\alpha} * \left(\frac{M}{P}\right)^{\gamma} * \left(\frac{P}{Power}\right)^{\delta+\epsilon} * \left(\frac{Power}{Space}\right)^{\epsilon}$$

P	Sustained Performance	$GFlop/s$	α
$\frac{M}{P}$	Effective Byte/Flop Ratio	$\frac{GByte}{GFlop/s}$	γ
$\frac{P}{Power}$	Power-efficiency	$\frac{GFlop/s}{kW}$	δ
$\frac{Power}{Space}$	Power-density	$\frac{kW}{m^2}$	ϵ

Double Your System! – But How?

Which system upgrade doubles your utility?

A: 2*P, 2*M, 2*Power (‘double system’)

B: 2*P, 2*M, 1*Power (Capability based – larger problems)

C: 2*P, 1*M, 1*Power (Capacity based – same problems)

D: 2*P, 2^k *M, 1*Power (Workload based – same time)

- The largest jobs possible complete in equal times.
- k represents workload scaling
- Linpack: $M \sim N^2$, $Flop \sim N^3$
  - k=2/3: $2^{2/3}$ *M = 1.59*M
- 0 <= k <= 1:
  k=0: C, capacity; k=1: B, ‘extreme’ capability

- Three (four) variables and (only) one condition means at least two (three) free parameters.

Double Your System! – But How?

$$UM(SC) = P^{\alpha} * \left(\frac{M}{P}\right)^{\gamma} * \left(\frac{P}{Power}\right)^{\delta+\epsilon} * \left(\frac{Power}{Space}\right)^{\epsilon}$$

Resulting conditions:

A: 2*P, 2*M, 2*Power => $\alpha = 1$

B: 2*P, 2*M, 1*Power => $\alpha+\delta+\varepsilon = 1$

C: 2*P, 1*M, 1*Power => $\alpha+\delta+\varepsilon-\gamma = 1$

D: 2*P, 2^{k} *M, 1*Power => $\alpha+\delta+\varepsilon-(1-k)\gamma = 1$

Generalized Utility Metric (Problem Scaled)

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D: Eliminate α; No 'space' (for now) => ε=0;

$$UM_{ps}(SC) = P^{1-\delta+(1-k)\gamma} * \left(\frac{M}{P}\right)^{\gamma} * \left(\frac{P}{Power}\right)^{\delta}$$

- For k=1 (extreme capability):
  - Memory exponent γ decouples!
  - Weighted geometric mean of Performance and Power-Efficiency.
- For k=0 (extreme capacity):
  - Almost (but not quite!) a weighted geometric mean between three quantities with 2 free weights.

CRD Generalized Utility Metric (Problem Scaled)

Pick D: What are the 'values' of other upgrades?

$$UM_{ps}(SC) = P^{1-\delta+(1-k)\gamma} * \left(\frac{M}{P}\right)^{\gamma} * \left(\frac{P}{Power}\right)^{\delta}$$

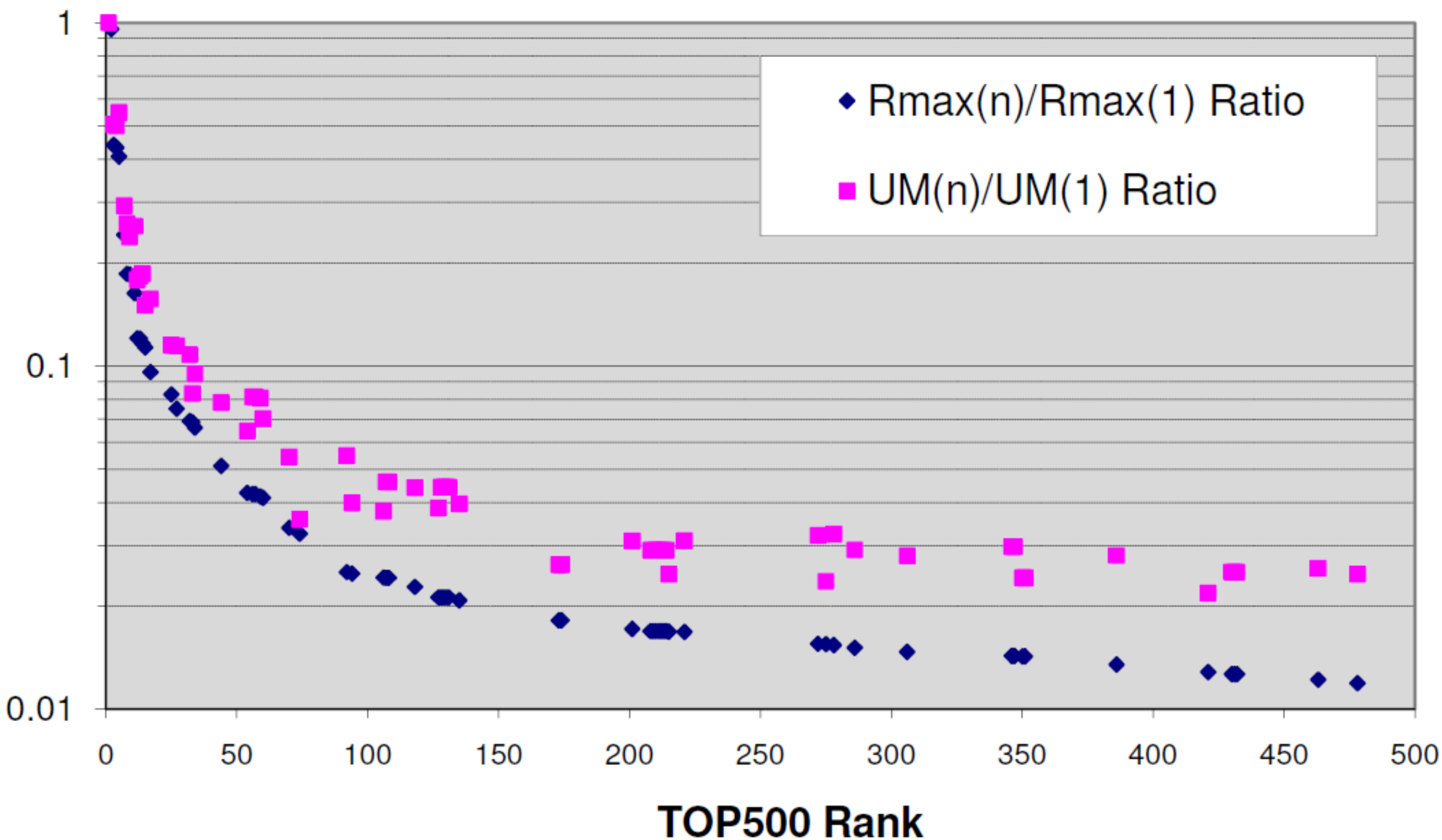
A (Double system): 2*P, 2*M, 2*Power => $2^{1-\delta+(1-k)\gamma}$

B (Replace system): 2*P, 2*M, 1*Power => $2^{1+(1-k)\gamma}$ ≥ 2

C (Upgrade processor): 2*P, 1*M, 1*Power => $2^{1-k\gamma}$ ≤ 2

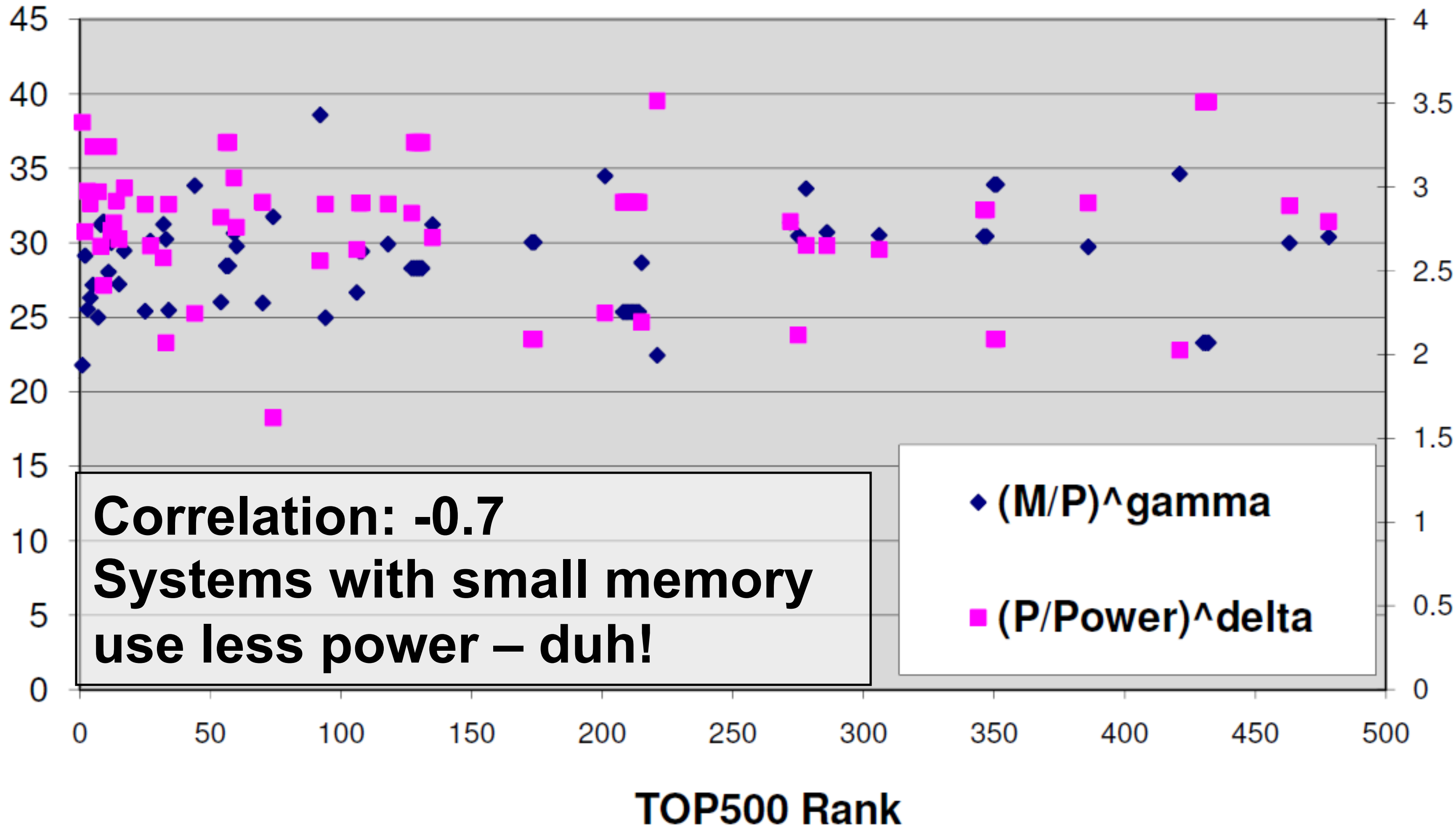
Ranking Changes

k=2/3; γ=0.2; δ=0.2; α=1-δ+(1-k)γ=0.8667



Ranking Changes

$k=2/3$; $\gamma=0.2$; $\delta=0.2$; $\alpha=1-\delta+(1-k)\gamma=0.8667$



Optimum System Sizes

- **Maximum of UM(SC)**
  - Power (and Space) grow linear with size

$$UM(n*SC) = P(n)^{\alpha} * \left(\frac{M(n)}{P(n)}\right)^{\gamma} * \left(\frac{P(n)}{Power(n)}\right)^{\delta}$$

- **Introduce Speedup S:** $S(n) = P(n)/P$
- **Differentiate:**
  d UM(n*SC) / d n

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Optimum System Sizes

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$$UM(n*SC) = \left(\frac{P(n)}{P}\right)^{\alpha-\gamma+\delta} * n^{\gamma-\delta} * P^{\alpha} * \left(\frac{M}{P}\right)^{\gamma} * \left(\frac{P}{Power}\right)^{\delta}$$

$$= S(n)^{\alpha-\gamma+\delta} * n^{\gamma-\delta} * UM(SC)$$

- **UM(n*SC) is given by Speedup and weights.**

Optimum System Sizes

- We differentiate UM(n*SC)

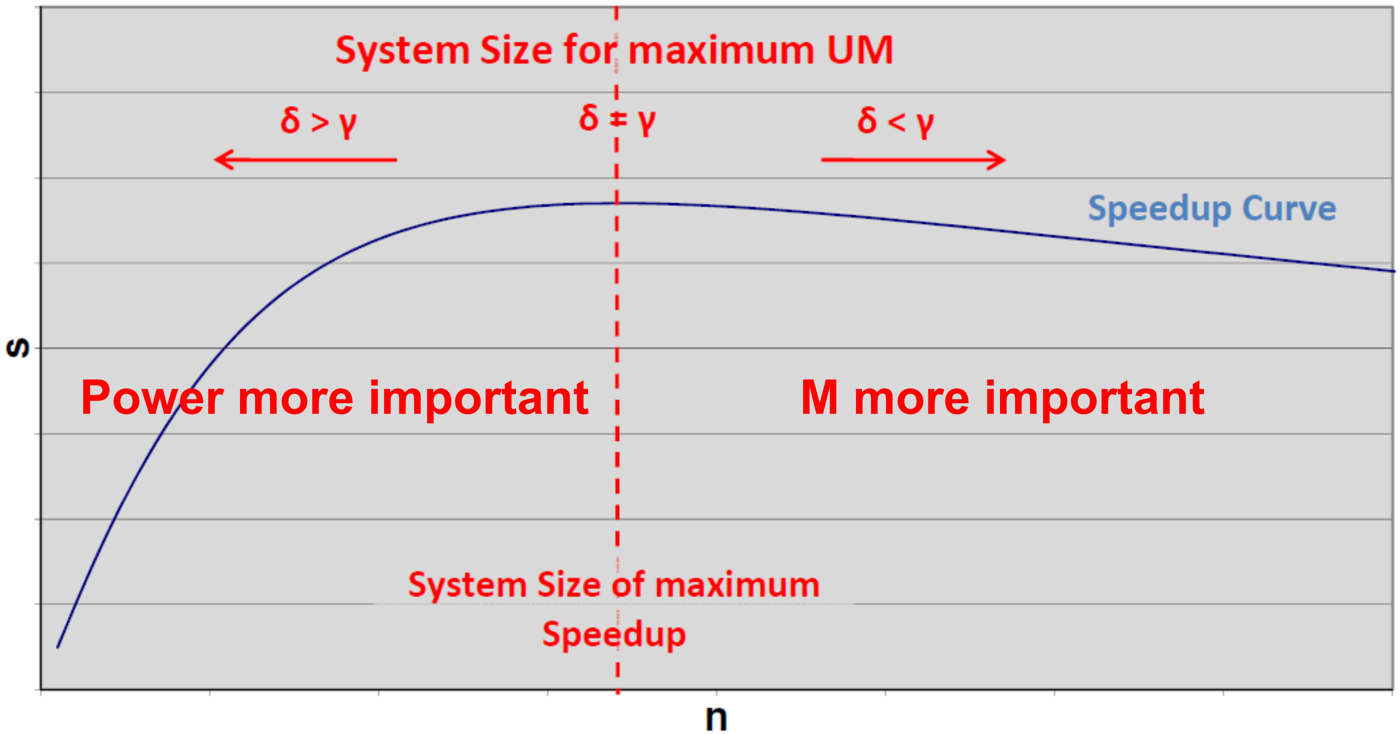
$$\frac{dUM(n*SC)}{dn} = \left((\alpha - \gamma + \delta) \frac{\frac{dS(n)}{dn}}{S(n)} + (\gamma - \delta)/n \right) * UM(n*SC)$$

Optimum System Sizes

- **Condition for optimum system size:**

$$(\alpha - \gamma + \delta) * \frac{dS(n)}{dn} * \frac{n}{S(n)} = \delta - \gamma$$

Maximum of UM relative to Speedup



Summary

- **Utility Metrics (UM) for ranking Supercomputers can be defined to include effects of memory sizes, power consumptions, and other factors.**
  - You have to decide how important these factors are.
- **Definition of an appropriate Performance metric still critical.**
- **UM is naturally formulated with factors:** Performance, Byte to Flop Ratio, Power efficiency, Power density.
- **(Partial) Calibration of parameters possible by normalizing the increase-rate of UM.**